Electron Screening in $^7Be + p \longrightarrow {}^8B + \gamma$ reaction

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(February 9, 2008)

We evaluate the effect of screening by bound electron in $(^7Be, e) + p \longrightarrow (^8B, e) + \gamma$ transition in the framework of the adiabatic representation of the three particle problem. Comparison with two approaches (united nucleus and static) is presented. We discuss possible applications of this effect both for Solar Neutrinos and low energy fusion experiments.

I. INTRODUCTION

In the recent years an increasing interest has been devoted to the accurate estimation of electron screening effects both for the solar plasma fusion rates and low energy Earth experiments (see Ref. [1–7] and references therein).

As a rule, one considers electrostatic screening in the solar plasma. This approach being classical or quantum correctly reflects the major properties of a process only for high relative velocity of the colliding nuclei, when electron density is supposed to be unchanged during the collision.

In the case, when relative velocity of the nuclei is too small in comparison with the electron one, the electron density is changed accordingly to any relative configuration of the nuclei. Thus the electrons have an impact on a kinetic energy shift of the nuclei. Such a process is considered in the adiabatic approach, which comes from the well known Born-Oppenheimer method.

An accurate treatment in the adiabatic approach of electrons from the continuum spectrum requires an additional research, while screening effects by bound electrons could be easily considered in the framework of the adiabatic approach (see, for example Ref. [8]).

For not very dense stars like the Sun, the commonly accepted Debye-Hückel approximation on the calculation of screening effect is not quite adequate. Recently this question was discussed by A. Dar and G. Shaviv [9]. This fact is usually of no importance considering that the screening due to the plasma electrons is by itself rather small (see Ref. [1,9]. On the other hand, the low-lying bound electrons on a nucleus do screen the electric charge of the colliding nuclei much effectively. Not only the stellar plasma but Earth experiments have such a feature.

Indeed, laboratory experiments are performed with atomic or molecular targets, and the ionic beam interacts with the target nuclei while the nuclei are surrounded by a number of bound electrons. It leads to that the fusion cross section is increased. At low and moderate energies, the fusion cross section of "bare" charged nuclei colliding with the relative momenta p and reduced mass M (in electron mass unit) is expressed as Ref. [10]:

$$\sigma(E) = \frac{S(E)}{E} e^{-2\pi\eta} \tag{1}$$

where $S(E)$ is the so-called astrophysical factor which incorporates all nuclear aspects of the process, E is the collision energy of nuclei and $\eta = MZ_1Z_2/a_e p$(the latter factor comes from $\psi_{coul}(0)$- the Coulomb wave function of the internuclear motion at the origin) and a_e is the hydrogen Bohr radius.

The screened cross section differs by a factor $\gamma(E)$ defined as:

$$\gamma(E) \equiv \frac{\sigma_{sc}}{\sigma_{ba}} = \frac{|\psi_E(0)|^2}{|\psi_{coul}(0)|^2} \tag{2}$$

where σ_{sc} and σ_{ba} are fusion cross sections both of the screened and "bare" nuclei, and $\psi_E(0)$ is the wave function of the internuclear motion at the origin, taking into account the bound electron around.

In this Letter we present the first quantum mechanical calculation of screening effect by bound electron in

$$(^7Be, e) + p \longrightarrow (^8B, e) + \gamma$$

nuclear fusion. This reaction is of no importance for the Sun luminosity but it is of great interest in Solar Neutrino Puzzle. An analysis of Solar Neutrino Experiment Data (see Ref. [11]) shows that there is no room for ^7Be neutrinos. At the same time ^8B neutrinos are presented. Since ^8B nuclei result from the reaction:

$$^7Be + p \longrightarrow {}^8B + \gamma, \tag{3}$$

it looks impossible to explain the existence of ^8B neutrinos and the absence of ^7Be neutrinos in the frame of the Standard Solar Model.

As one of the main consequences of electron screening effect applied to all solar fusions could be decrease in the Sun core temperature. Increased values of fusion cross sections could lead to the Sun cooling with the same observable Solar luminosity. However, just cool Sun models do not solve the Solar Neutrino Problem (see Ref. [12]).

We show that bound electron essentially enhances the fusion rate in comparison with the reaction (3) rate. Physics of this phenomena could be easily understood in the framework of united nucleus approach (see below). "Exact" solution of our problem is obtained in adiabatic approach for three particle problem. We compare our solution with the two relevant approaches (united nucleus and static approaches, described below) which give upper and lower estimate for the screening effect, in the considered here three-particle picture.

A finite $^7Be - p$ interaction radius also is of great importance. A larger radius results in higher tunneling probability into the internal region and thus to a higher $^7Be(p,\gamma)^8B$ cross section. Theoretical model Ref. [13] also predicts increased astrophysical $S_{17}(0)$-factor. All above mentioned effects result in the increased ^8B production rate and the latter could be a really alternative to $^7Be + e \longrightarrow {}^7Li + \nu_e$ reaction.

II. METHOD OF CALCULATION

We treat the Coulomb problem for three particles in the framework of the adiabatic representation. The basic two-center eigenfunctions are derived from the Schrödinger equation for two nuclei with electric charges Z_1 and Z_2 (in electron charge unit) ($Z_1 > Z_2$) fixed on a distance R and for an electron around them. The Schrödinger equation is transformed into a infinite system of equations with separated variables. Our approximation consists in that we use only one two-center eigenfunction corresponding to the ground state of the system. There are some arguments for this approach.

At first, the high energy states corrections (at fixed R) are of order of magnitude about the ratio of the electron mass to the proton mass.

Then, excited energy levels correspond to the less energy of the united nucleus. It leads to their exponentially small contribution into the nuclear fusion rate in comparison with the ground state of the system.

At least, only the ground state energy of the electron in the field of ^7Be and p nuclei, called $1S\sigma$ therm (see, for example Ref. [14]) has the correct asymptotic behaviour (see below).

The three particle wave function is presented as:

$$\Psi(\vec{R},\vec{r}) = \phi(\vec{R}) \cdot \psi(\vec{R},\vec{r}),$$

where $\phi(\vec{R})$ is the wave function of two colliding nuclei and $\psi(\vec{R},\vec{r})$ is the electron wave function, which depends on the internuclear distance $\vec{R}$.

The electron energy eigenvalue $U_{nlm}(R)$ also depends on the internuclear distance. We use the tabulated values of U_{nlm} from Ref. [15] for our purposes.

In Fig. 1 the values for the ground state are plotted. At $R \to 0$ the electron energy approaches to the energy of the united ion: $U \to -\frac{(Z_1+Z_2)^2}{2}$ (energy unit here is 27.21 eV) and for $R \to \infty$ the electron energy approaches to the energy of the isolated ion eZ_1: $U \to -\frac{Z_1^2}{2}$.

We consider the case $Z_1 = 4$ and $Z_2 = 1$. In the adiabatic approach the electron energy $U_{nlm}(R)$ in the field of two nuclei serves as an effective attraction potential for the nuclei. Evidently the main effect of the electron screening comes from the collisions with zero orbital moment of nuclei. The latter and the correct behaviour of $U_{nlm}(R)$ (at $R \to 0$ and $R \to \infty$) for the ground state allow us to consider the case $n = l = m = 0$. Let us denote U_{000} as $U(R)$. With the electron energy $U(R)$ in hand one can solve the scattering problem for two nuclei ^7Be and p. The total effective potential reads:

$$V(R) = \frac{Z_1 Z_2}{R} + U(R) \tag{4}$$

We calculate values of the wave function of relative motion of the two nuclei at kinetic energies 0.1 keV $\leq E \leq$ 100 keV and compare the calculated $|\Psi(0)|^2$ with $|\Psi_{coul}(0)|^2 = \frac{2\pi\eta}{e^{2\pi\eta}-1}$ - value of the scattering wave function of two particles in the origin.

For simple estimations one can use well known united nucleus approach that consists in following. Consider a fusion of two nuclei from initial states with bound electrons. Let the total negative energy of these bound electrons be $E_1 + E_2$. In the final state there is an united nucleus with bounded electrons around and let the total energy of the electrons in this new ion be E_u. Thus the difference $\triangle E = E_1 + E_2 - E_u$ adds to the kinetic energy of the two nuclei

accelerating them. So, the united nucleus approach changes $|\Psi_E(0)|^2$ to $|\Psi_{E+\triangle E}(0)|^2$. This is a rather good approach since the main contribution to the value $|\Psi_E(0)|^2$ comes from internuclear distances less then classical return point in the potential.

Since united nucleus approach replaces decreasing $E(R)$ (see in Fig. 1) by the constant $\triangle E$ this approach serves as an upper estimate.

Another approach uses static wave function of the bound electron. It means that this wave function does not depend on the internuclear distance. Total three particle wave function is presented as a product:

$$\Psi_s(\vec{R},\vec{r}) = \phi_s(\vec{R}) \cdot \psi_s(\vec{r}),$$

where the wave function for the electron bounded on the nucleus with electric charge Z_1 reads: $\psi_s(\vec{r}) = \sqrt{\frac{Z_1^3}{\pi}} e^{-Z_1 r}$, and $\phi_s(\vec{R})$ is the wave function of two nuclei. Averaging the electron coordinates , $\phi_s(\vec{R})$ can be derived from the Schrödinger equation with the potential:

$$V_{st}(R) = \frac{Z_1 Z_2}{R} - Z_2 \cdot \left(\frac{1 - e^{-2RZ_1}}{R} - Z_1 e^{-2RZ_1} \right). \tag{5}$$

Kinetic energy of the colliding nuclei is counted from the electron eigenvalue energy in the field of Z_1 nucleus. This approach works well for nuclei with relative velocity v much greater then electron velocity v_e: $v >> v_e$.

Attractive part of the potential V_{st} approaches to the value :$Z_1 Z_2$ (in 27.21 eV unit) at $R \longrightarrow 0$. Since $Z_1 Z_2 < \frac{(Z_1+Z_2)^2}{2}$-the value of the electron energy in united ion, the static approach gives a lower estimate for an enhancement factor.

We use all three approximations in order to compare their validity.

III. RESULTS

The numerical solution of the scattering problem of the two nuclei in the potential (4) was obtained on the mesh on R for $R = 0$ and $R = R_{max}$ in Numerov's scheme. We varied step h and R_{max} in order to ensure that final result is not changed substantially. Also for checking purposes we reproduced the value of $|\Psi_{coul}(0)|^2$ substituting $U(R) = 0$.

The enhancement factor (2) is plotted in Fig. 2. In Fig. 3 we compare the "exact" numerical solution with both united nucleus and static approaches.

At kinetic energies above 2 keV a very good agreement between all three approaches is obtained, though at lower energies the united nucleus approach overestimates and the static approach underestimates an electron screening effect. It is easy to see that simple united nucleus prescription gives much closer values for electron screening effect then the static approach. Thus the united nucleus approach being compared with "exact" numerical solution could be easily used in stellar evolution code.

As one can see, electron screening could essentially increase nuclear fusion rate and must be taken into account both for Earth experiments at low energies and for an understanding of stellar phenomena.

Deficit of ^7Be neutrinos in comparison with ^8B neutrinos (called "second Solar Neutrino Problem", see Ref. [16]) could be a good example of this effect. At the same Solar Model input data the flux of ^8B neutrinos increases due to the increased rate of ^8B nuclei production and the flux of ^7Be neutrinos decreases due to the alternative reaction (3). At mean kinetic energy in the Sun core at temperature $T = 15T_6$ ($T_6 = 10^6$K) this effect is 15 times of magnitude.

However due to the Gamov's "window" around $17 - 20$ keV the screening effect of the bound electron is reduced to 10%. But if some nuclear fusion reaction go faster in the Sun core (due to the electron screening) then the temperature in the Sun core could be lower. Under this assumption it might be possible to explain the Solar Neutrino Problem.

In conclusion, we have performed quantum mechanical calculation of the screening effect taking place in $(^7Be, e) + p \longrightarrow (^8B, e) + \gamma$ reaction at finite kinetic energy of the nuclei. We compared our results with two approaches giving upper and lower limits for the screening effect and studied the validity of the approaches. Possible application to the Solar Neutrinos is briefly discussed.

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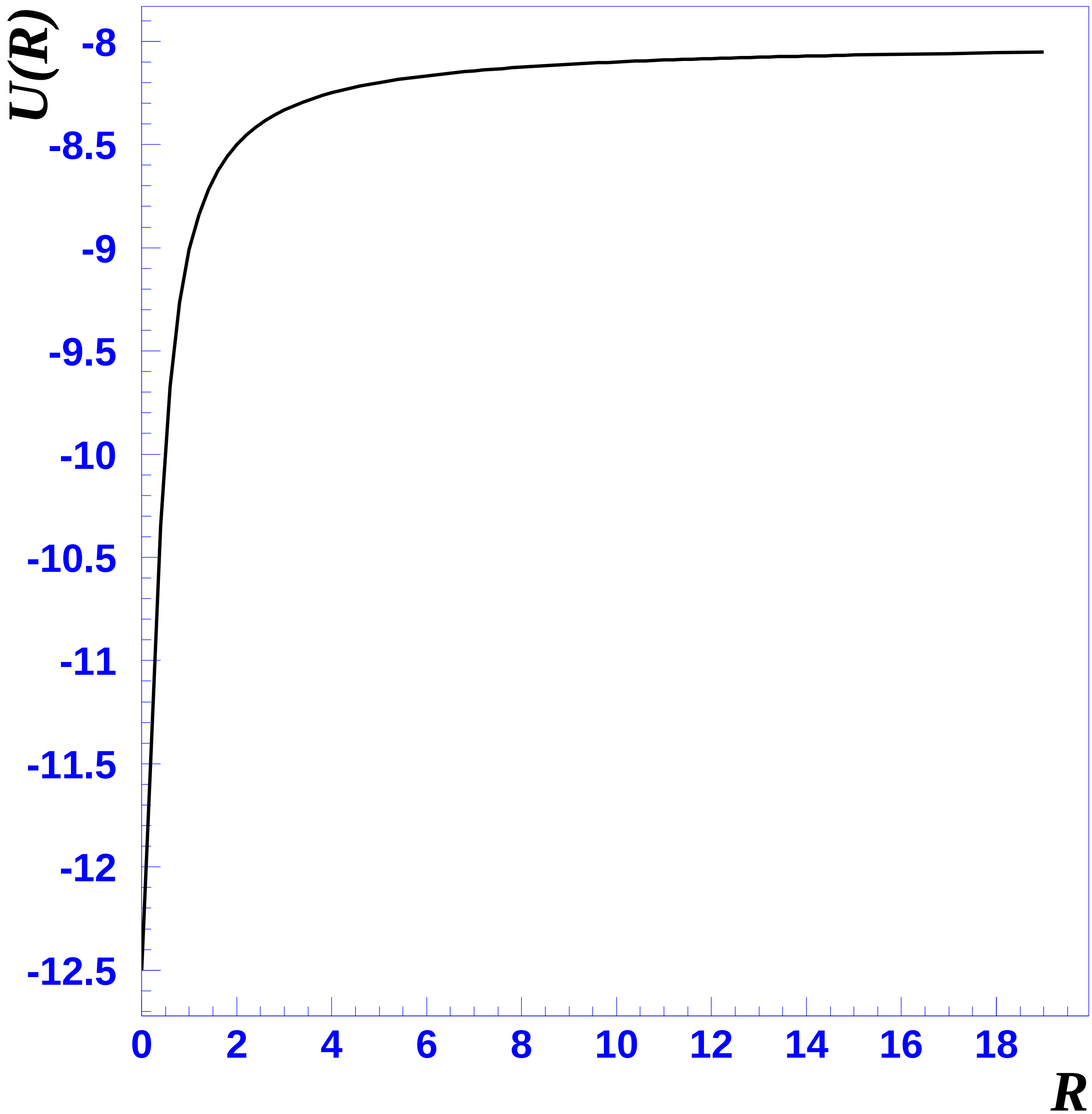

FIG. 1. Electron energy in the field of the two nuclei ^7Be and p. R measured in Bohr radius, $U(R)$ given in 27.21 eV unit

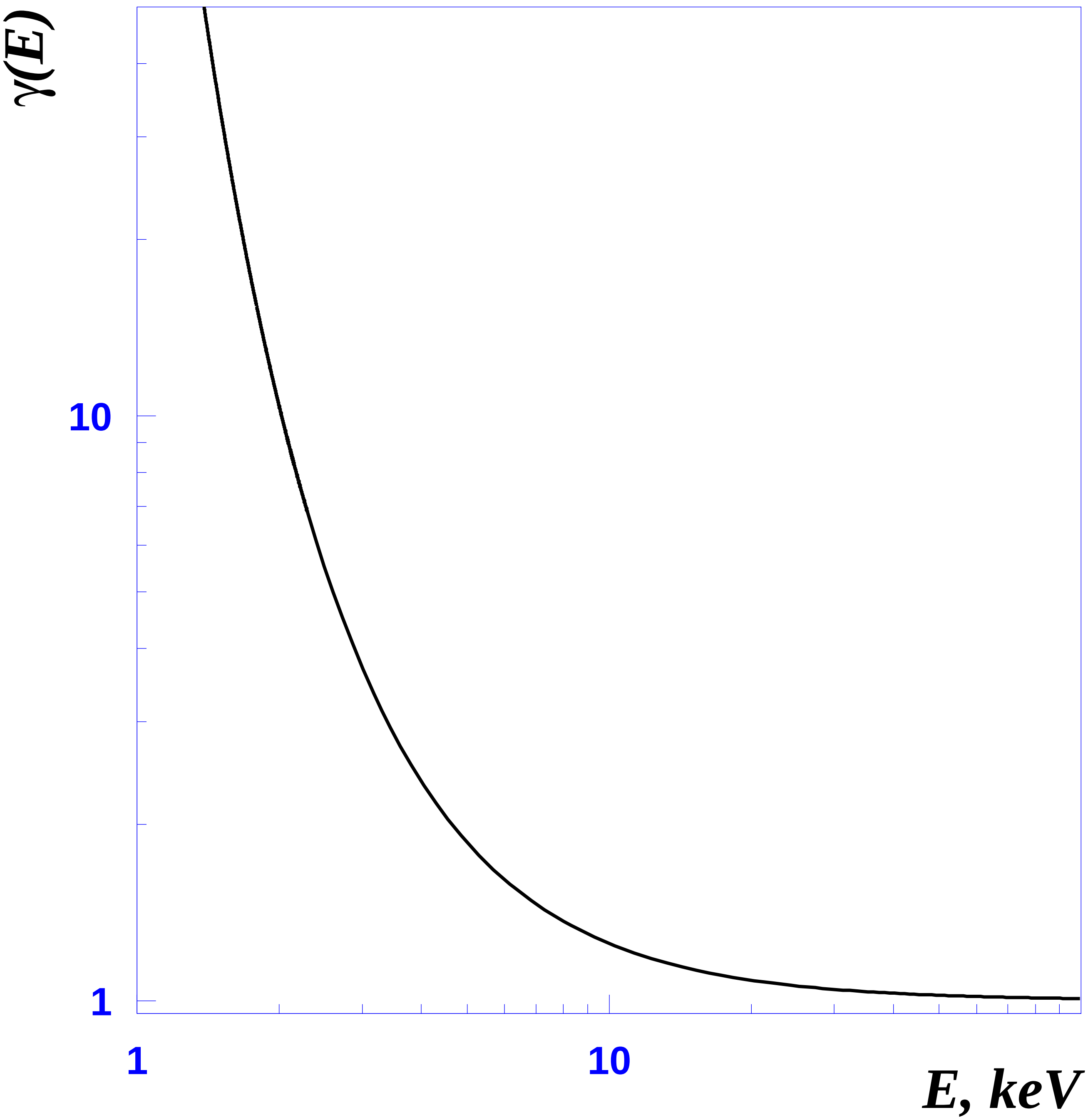

FIG. 2. Enhancement of nuclear fusion rate due to the electron screening

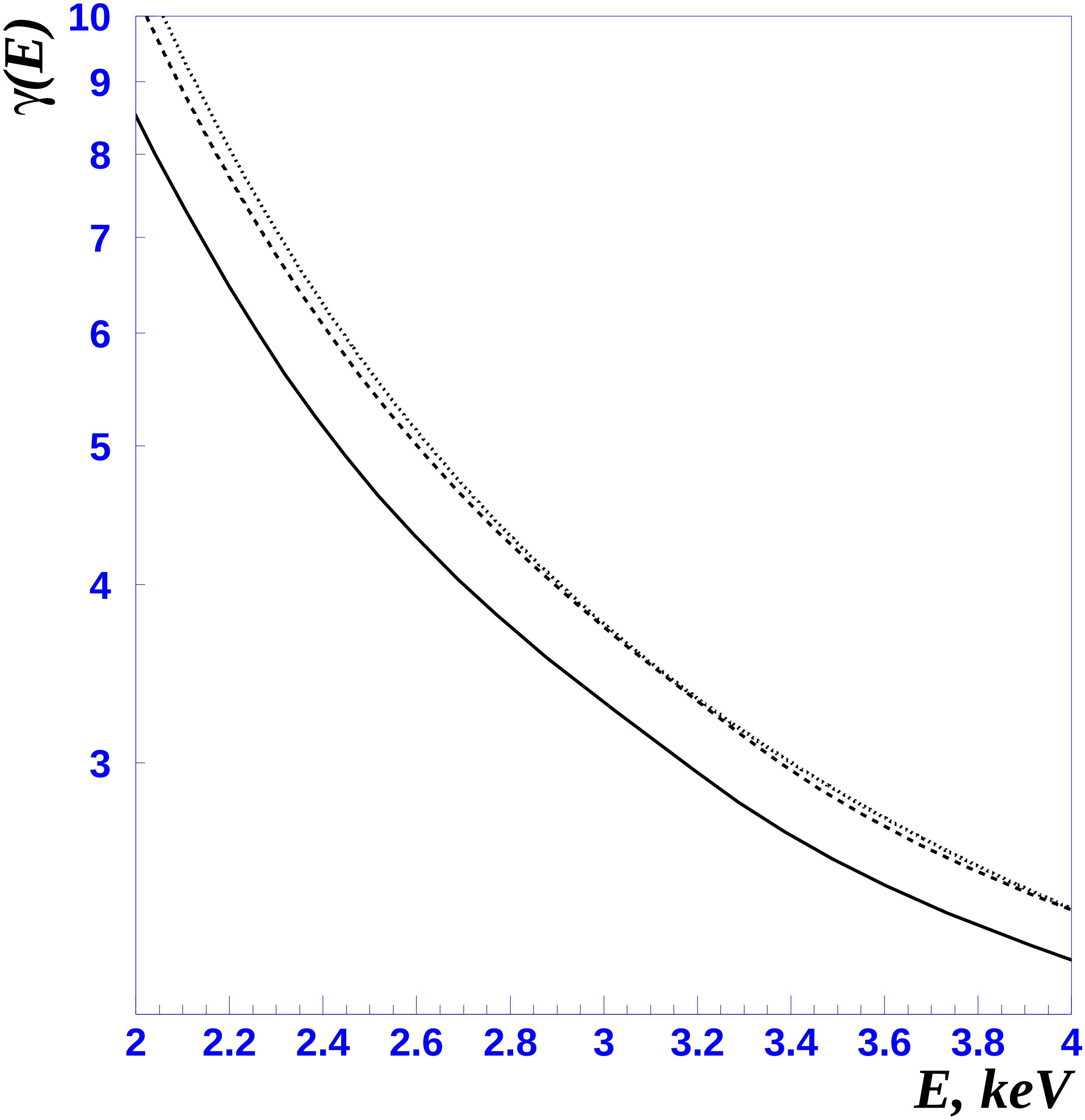

FIG. 3. $\gamma(E)$ factor for united nucleus (dotted line), "exact" numerical solution (dashed line) and static approaches (solid line)